On The Surd Equation $\sqrt{2z} = \sqrt{x+a\,y} + \sqrt{x-a\,y}\ \ (a \neq 0)$

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***Abstract*: In this short paper, non-zero integer distinct integer solutions to the surd equation with three unknowns given by $\sqrt{2z} = \sqrt{x+a\,y} + \sqrt{x-a\,y}, (a \neq 0)$ are obtained through the integer solutions of Pythagorean equation.**

***Keywords*: Surd equation, transcendental equation, integer solutions.**

1. Introduction

Diophantine equations have an unlimited field of research by reason of their variety. Most of the Diophantine problems solved by the researchers are algebraic equations [1], [2].

It seems that much work has not been done in finding the integer solutions to transcendental equations involving surds. In this context, refer [3]-[18] to the integral solutions of transcendental equations involving surds. This short communication analyses a transcendental equation with three unknowns given by, $\sqrt{2z} = \sqrt{x+a\,y} + \sqrt{x-a\,y}$.

Infinitely many non-zero integer triples (x, y, z) satisfying the above equation are obtained through employing the integer solutions to the well-known Pythagorean equation.

2. Method of Analysis

The surd equation to be solved is,

$$\sqrt{2z} = \sqrt{x+a\,y} + \sqrt{x-a\,y}\ (a \neq 0) \tag{1}$$

On squaring both sides of (1), it simplifies to,

$$z = x + \sqrt{x^2 - a^2 y^2} \tag{2}$$

To eliminate the square-root on the R.H.S. of (2), take,

$$x^2 - a^2 y^2 = \alpha^2 \tag{3}$$

which is in the form of the well-known Pythagorean equation,

$$X^2 + Y^2 = Z^2 \tag{4}$$

Employing the most cited solutions of (4), observe that (3) is satisfied by,

$$x = a^2 r^2 + s^2\ , y = 2rs \tag{5}$$
$$\alpha = a^2 r^2 - s^2\ , r \geq s \geq 0$$

In view of (2), it is seen that,

$$z = 2a^2 r^2 \tag{6}$$

Thus, (5) and (6) represent the integer solutions to (1).

A few numerical solutions are presented in Table 1 below,

Table 1
Numerical solutions

a	r	s	x	y	z
1	2	1	5	4	8
2	3	2	40	12	72
3	5	3	234	30	450

It is worth mentioning that, (3) is also satisfied by

$$x = a^2(r^2 + s^2)\ , y = a(r^2 - s^2)\ , r \geq s \geq 0 \tag{7}$$

$$\alpha = 2a^2 rs$$

From (2), the value of z is given by

$$z = a^2 (r+s)^2 \tag{8}$$

Thus, (7) and (8) satisfy (1).

A few numerical solutions are presented in Table 2 below

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Table 2
Numerical solutions

a	r	s	x	y	z
1	2	1	5	3	9
2	3	2	52	10	100
3	5	3	306	48	576

Further, (3) is also satisfied by,

$$x = a(m^{s+2} + m^s)\ ,\ y = 2m^{s+1} \qquad (9)$$
$$\alpha = a(m^{s+2} - m^s)$$

From (2), the value of z is given by,

$$z = 2am^{s+2} \qquad (10)$$

Thus, (9) and (10) satisfy (1).

A few numerical solutions are presented in Table 3 below,

Table 3
Numerical solutions

a	m	s	x	y	z
1	2	0	5	4	8
2	3	2	180	54	324
3	5	3	9750	1250	18750

3. Conclusion

In this paper, we have presented integer solutions to the surd equation with three unknowns given by,

$$\sqrt{2z} = \sqrt{x+ay} + \sqrt{x-ay}\ (a \neq 0)$$

To conclude one may attempt to find integer solutions to other choices of surd equations. With unknowns three or more than three.

The above section says how to prepare a subsection. Just copy and paste the subsection, whenever you need it. The numbers will be automatically changes when you add new subsection. Once you paste it, change the subsection heading as per your requirement.

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